647510 **BSc6MatC602x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
April/May-2022 (NEW COURSE)
Mathematical Analysis - 2 and Abstract Algebra-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

QUE. 1 (A) Answer any two out of three. **[10]**

(1) Show that continuous image of compact set is compact.

(2) State and prove theorem of nested interval.

(3) Show that compact subset of metric space is closed and bounded.

(B) Answer any one out of two. **[4]**

(1) Prove that non empty subset of discrete metric space is singleton iff it is connected.

(2) (i) Check whether set A={1,16,49,……..} is countable.

(ii) Check whether set $E=(1,3]\cap[3,5)$ is connected.

QUE. 2 (A) Answer any two out of three. **[10]**

(1) if f(t) is continuous for all $t \geq 0$ and f'(t) is piecewise Continuous then prove that L{f'(t)} Exists,

Provided $\lim\limits_{t\to\infty}[e^{-st}f(t)]=0$ and

$L\{f'(t)\}=sL\{f(t)\}-f(0)=s\,\overline{f}(s)-f(0)$. Also find L[sin(2t) cos(3t)].

(2) State and prove change the scale property. Using this property find L[sinh(at) sin(at)].

(3) Find Laplace transform of

$f(t) = \sin(t)$ where $0 < t < \pi$

$= 0$ where $t > \pi$

(B) Answer any one out of two. **[4]**

(1) Find : $L^{-1}\left[\dfrac{s+7}{s^2+2s+2}\right]$.

(2) Find : $L^{-1}\left[\dfrac{s+1}{(s^2+4)s^2}\right]$.

QUE. 3 (A) Answer any two out of three. **[10]**

(1) Using convolution theorem Find $L^{-1}\left[\dfrac{s}{(s+1)(s^2+1)}\right]$.

(2) Prove that if $L\{f(t)\}=\bar{f}(s)$ then $L\left[\int_0^t f(u)du\right]=\dfrac{1}{s}\bar{f}(s)$.

Using this result Find $L^{-1}\left[\dfrac{1}{s(s+a)^3}\right]$.

(3) if $L\{f(t)\}=\bar{f}(s)$ and $\dfrac{f(t)}{t}$ has laplace transform then

Prove That $L\left\{\dfrac{f(t)}{t}\right\}=\int_s^\infty \bar{f}(s)ds$. Using this result Find

$L\left[\dfrac{\cos(2t)-\cos(3t)}{t}\right]$.

(B) Answer any one out of two. **[4]**

(1) By using integration of Laplace transform find $L[\frac{1-e^{t}}{t}]$.

(2) Find : $L[e^{-4t}\int_0^t t\sin(3t)dt]$.

QUE. 4 (A) Answer any two out of three. **[10]**

(1) Prove that if $K \le G'$ then $\phi^{-1}(K) \le G$. where $(G,*)$ and $(G',*')$ be two groups and $\phi : G \to G'$ is Homomorphism.

(2) Show that every field is an integral domain.is converse true ? Explain with example.

(3) State and Prove necessary and sufficient condition for non- empty Subset of ring to be a subring.

(B) Answer any one out of two. **[4]**

(1) Define: Homomorphism, Kernal of Homomorphism, Zero divisor, integral domain.

(2) (C_0, x) and $(R, +)$ be two groups. $f : R \to C_0$ define as ; for $x \in R$, $f(x)=e^{ix}$. Show that f is homomorphism and find K_f.

QUE. 5 (A) Answer any two out of three. **[10]**

(1) State and prove division algorithm theorem of Polynomial.

(2) For non-zero polynomial $f(x),g(x) \in D[x]$, prove that $\deg(fg)=\deg(f)+\deg(g)$.

(3) Show that $f(x) = x^4-2x^2+8x+1$ is irreducible over Q.

(B) Answer any one out of two. **[4]**

(1) Find G.C.D. d(x) of $f(x),g(x) \in Z_5[X]$. where $f(x)=x^3+3x^2+3x+3$ and $g(x)=4x^3+2x^2+2x+2$. Also express d(x) as a(x)f(x)+b(x)g(x).

(2) if $a=a_1+a_2i+a_3j+a_4k$ is non-zero quaternion number then find a^{-1}. Also check whether quaternion set is field.

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